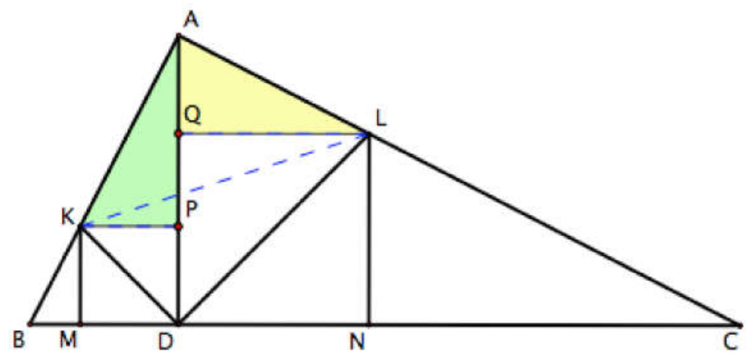
Thus,

$$\frac{KM}{AD} + \frac{NL}{AD} = 1,$$

and $KM + NL = AD$, as desired.



Solution 2, a composite of similar solutions by Sefket Arslanagić and Joel Schlosberg.

Let P and Q be the feet of the perpendiculars to AD from K and L, respectively. Then $KMDP$ and $LNDQ$ are rectangles (since each has three angles known to be $90°$) ; in fact, they are squares because the diagonals from D bisect the right angles there. Thus,

$$KM = KP \quad \text{and} \quad LN = QD. \tag{1}$$

Note that also $\angle LDK = 90°$ (because DK and DL both bisect the right angles formed by AD at D). It follows that $AKDL$ is cyclic (because of the right angles at A and D), whence

$$\angle LKA = \angle LDA = \frac{1}{2}\angle CDA = 45°,$$

and we deduce that triangle AKL, with its right angle at A, is isosceles with congruent legs AK and AL. Since $\angle LQA = 90° = \angle APK$, and $\angle QAL = 90° - \angle KAP = \angle PKA$, it follows that triangles AQL and KPA are congruent. Thus $KP = AQ$; but we know from (1) that $KM = KP$ and $NL = QD$. We have, therefore,

$$KM + NL = KP + QD = AQ + QD = AD$$

as desired.

3863. *Proposed by Michel Bataille.*

Let a, b, c be real numbers such that $a^2 + b^2 + c^2 \leq 1$. Prove that

$$a^2b(b-c) + b^2c(c-a) + c^2a(a-b) \geq \frac{(b-c)^2(c-a)^2(a-b)^2}{2}\ .$$

We received ten correct submissions. We present the identical solution by Alkady Alt and Adnan Ali, done independently.

Since $a^2 + b^2 + c^2 \leq 1$, we have by Cauchy-Schwarz inequality that

$$\begin{aligned}
&2(a^2b(b-c) + b^2c(c-a) + c^2a(a-b)) \\
&\quad = (ab-bc)^2 + (bc-ca)^2 + (ca-ab)^2 \\
&\quad \geq (b^2+c^2+a^2)((ab-bc)^2 + (bc-ca)^2 + (ca-ab)^2) \\
&\quad \geq (b(ab-bc) + c(bc-ca) + a(ca-ab))^2 \\
&\quad = (ab^2 + bc^2 + ca^2 - a^2b - b^2c - c^2a)^2 \\
&\quad = ((a-b)(b-c)(c-a))^2,
\end{aligned}$$

which completes the proof.

3864. *Proposed by Cristinel Mortici.*

For every positive integer m, denote by $m!!$ the product of all positive integers with same parity as m, which are less than or equal to m. Let $n \geq 1$ be an integer. Prove that

$$(-1)^n(2n)!! - (2n-1)!! + (2n+1)!!\sum_{k=1}^{n} \frac{1}{2k-1}$$

is divisible by $(2n+1)^2$.

One incorrect solution was received. We present the solution of the proposer.

Let

$$p(x) = (x-1)(x-3)\dots(x-(2n-1)).$$

Let n be even. Then

$$p(x) = x^2q(x) - (2n-1)!!\sum_{k=1}^{n} \frac{1}{2k-1}x + (2n-1)!! \qquad (1)$$

for some polynomial $q(x)$. For $x = 2n+1$, we get

$$(2n)!! = (2n+1)^2q(2n+1) - (2n+1)!!\sum_{k=1}^{n} \frac{1}{2k-1} + (2n-1)!!,$$

$$(2n)!! - (2n-1)!! + (2n+1)!!\sum_{k=1}^{n} \frac{1}{2k-1} = (2n+1)^2q(2n+1).$$

If n is odd, then (1) becomes

$$p(x) = x^2q(x) + (2n-1)!!\sum_{k=1}^{n} \frac{1}{2k-1}x - (2n-1)!!$$

and the conclusion follows by again taking $x = 2n+1$.